বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2022

(Under CBCS Pattern)

Semester - II

Subject : MATHEMATICS

Paper : C 3 - T

[REAL ANALYSIS]

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any ***five*** questions : 2×5=10

(a) Define least upper bound of a bounded set and obtain it for the set

$$A=\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},......\frac{n}{n+1},.....\right\}$$

(b) Define point of accumulation of a set and find all the points of accumulation of the set $E=\left\{\frac{1}{m}+\frac{1}{n}/\,m,n=1,2,3,.....\right\}$.

(c) Prove that if A and B are two closed sets, then $A\cup B$ and $A\cap B$ are both closed sets.

(d) Show that $\left\{\dfrac{3n+1}{n+2}\right\}$ is a bounded sequence.

(e) Prove that every convergent sequence is bounded. Is the converse true? Justify.

(f) Prove that the Series $\sum_{n=1}^{\infty}\dfrac{1}{n(n+1)}$ converges.

(g) If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.

(h) Define compact set with an example.

2. Answer any ***four*** questions : 5×4=20

(a) Define countability of a set. Show that the set of all real numbers is not countable.

(b) State and prove Archimedean property of real numbers.

(c) If a set S is open, then prove that its complement is a closed set. Is the converse true? Justify.

(d) Define Cauchy Sequence. Prove that the sequence $\{n^2\}$ is not a Cauchy Sequence.

(e) Prove that every bounded sequence has a convergent subsequence.

(f) Prove that $1+\dfrac{1}{2!}+\dfrac{1}{4!}+\dfrac{1}{6!}+.......$ converges.

3. Answer any ***three*** questions : 10×3=30

(a) What do you mean by convergence, absolute convergence and conditional convergence of a series of real numbers? Prove that absolutely convergence imply convergence. Classify as to divergent, conditionally convergent or absolutely of the following series :

(i) $1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}+.....$

(ii) $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.....$

(iii) $\frac{1}{2}-\frac{2}{3}+\frac{3}{4}-\frac{4}{5}+.....$ 3+1+6

(b) If a sequence $\{x_n\}$ of real numbers is monotonic increasing and bounded above, then prove that it converges to its exact upper bound. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is monotonic increasing and bounded above. 5+5

(c) (i) State and prove Bolzano-Weierstrass theorem for sequences.

(ii) Using Cauchy's general principle of convergence prove that $\{x_n\}$, where $x_n=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....+(-1)^{n-1}\cdot\frac{1}{n}$, is a convergent sequence. 5+5

(d) (i) State and prove Heine-Borel theorem. Give an illustration which justify Heine-Borel theorem.

(ii) State and prove density property of real numbers. 4+3+3

(e) (i) Examine if the following series converge :

(i) $\sum_{n=1}^{\infty}\frac{n+1}{10^{10}(n+2)}$ (ii) $\sum_{n=1}^{\infty}\frac{n+1}{n+2}$ (iii) $\sum_{n=1}^{\infty}\log\left(1+\frac{1}{n}\right)$

(ii) Given $0<x_1<x_2$. If each $x_n=\frac{x_{n-1}+x_{n-2}}{2}$,

then prove that $\{x_n\}\to\frac{1}{3}(x_1+2x_2)$ as $n\to\infty$. 6+4

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Semester - II

Subject : MATHEMATICS

Paper : C 4 - T

[DIFFERENTIAL EQUATIONS & VECTOR CALCULUS]

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any ***five*** questions : 2×5=10

(a) What do you mean by the indicial equation?

(b) What is the phase plane?

(c) If $f_1, f_2, ...f_m$ are solution of mth order linear homogeneous differential equation, then show that $c_1f_1 + c_2f_2 + ... + c_mf_m$ is also a solution of this equation.

(d) Transform $x^3 \frac{d^3y}{dx^3} + y = 0$ into the differential equation with constant co-efficients.

(e) Explain Wronskian and its properties.

(f) Define a space curve and its tangent.

(g) Evaluate $\int \overline{A} \times \frac{d^2\overline{A}}{dt^2} dt$.

(h) Evaluate : $\frac{1}{D^2 - 1} 4xe^x$ where $D \equiv \frac{d}{dx}$.

2. Answer any ***four*** questions : 5×4=20

(a) Solve $z^2 \frac{d^2 y}{dz^2} - 3z\frac{dy}{dz} + y = \frac{\log z \sin(\log z) + 1}{z}$.

(b) Solve the following initial value problem by using the method of undetermined co-efficients $\frac{d^2 y}{dx^2} - 8\frac{dy}{dx} + 15y = 9xe^{2x},\ y(0) = 5,\ y'(0) = 10$.

(c) Suppose $\overline{A} = x^2 yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$ and $\overline{B} = 2z\hat{i} + y\hat{j} - x^2\hat{k}$.

Find $\frac{\partial^2}{\partial x \partial y}(\overline{A} \times \overline{B})$ at (1, 0, –2).

(d) Develop the method of variation of parameter in connection with the general second order linear differential equation with variable coefficients

$$a_0(x)\frac{d^2 y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x) y = F(x).$$

(e) Solve the initial value problem : $\frac{dx}{dt} = -2x + 7y,\ \frac{dy}{dt} = 3x + 2y;\ x(0) = 9$ and $y(0) = -1$.

(f) Solve : $(D^2 + 2) y = x^2 e^{3x} + e^x \cos 2x$.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Find the solution of the equation $\frac{d^2x}{dt^2} - x = 2$, which satisfies the conditions $\frac{dx}{dt} = 3$ when $t = 1$ and $x = 2$ when $t = -1$. 8

(ii) Define the stable equilibrium. 2

(b) (i) Find the power series solution in power of x of the following differential equation $3x\frac{d^2y}{dx^2} - (x-2)\frac{dy}{dx} - 2y = 0$. 8

(ii) State Lipschitz condition for a function $f(x, y)$ on D. 2

(c) (i) Find the equation of the tangent plane to the surface $x^2 + 2xy^2 - 3z^3 = 6$ at the point P (1, 2, 1). 5

(ii) Find the work done in moving a particle by the force field $\overline{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the curve defined by $x = 2t^2,\ y = t,\ z = 4t^2 - t$ from $t = 0$ to 1. 5

(d) (i) Given that $y = e^{2x}$ is a solution of $(2x+1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0$, find the linearly independent solution by reducing the order. Write the general solution. 7

(ii) Write down the solution of $\frac{d^4y}{dx^4} - 3\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 12y = 0$. 3

(e) (i) Find the power series solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ in powers of $(x-1)$. 6

(ii) Solve $\frac{d^4y}{dx^4} + y = \cos h(4x)\sin h(3x)$. 4